A NEW VARIABLE STAR.

By Torvald Köhl.

The star No. 121 in Birmingham's Catalogue, = No. 144 in Chandler's Catalogue of red stars,—position for 1875.0: $5^h\ 38^m\ 12^s.47\ (+3^s.57)$, $+20^\circ\ 38'\ 24''.9\ (+1''.9)$—has shown a remarkable change in brightness. It has formerly been estimated as a star of the 7.5th magnitude (B. D. has 7.7, Berlin A. G. Catalogue has 7.2). Dreyer observed it at Dublin from 1875 to 1879, and I at Odder from 1887 to 1893, without seeing any change of light in this orange-red star until on January 22, 1898, when I was surprised at the faintness of the star, which is now of about the 9th magnitude, and thus it has also been seen on the dates January 27 and 31 and February 1, 1898.

Odder, Denmark, February 6, 1898.

MAGNIFYING RATIOS OF EWING SEISMOGRAPHS OF THREE COMPONENTS, AND OF THE DUPLEX-PENDULUM SEISMOGRAPHS.

By C. D. Perrine.

In the following deductions the pen and plate are assumed to move with respect to the steady-point, and the motions of each are considered separately. In the reduction of the recorded displacements given by the pens upon the smoked glass plate, to the actual displacement of the Earth particle, there are several circumstances to be taken into account. In the case of the two horizontal components there are four considerations, viz:—

A.—The ratio of the pens, *i. e.* the distance from the point of the pen to the steady-point, divided by the distance from the steady-point to the point of support.

B.—The angle which the meridian of the pens makes with the true meridian of the place. If they coincide, there is no factor to be introduced on that account.

C.—The angle which a radius of the circular plate drawn through the point of the pen makes with a line drawn through

the point of the pen and the steady-point. If this angle is ninety degrees, there is no factor to be introduced on this account.

D.—The effect on the record caused by the motion of the record-plate itself, due to the earthquake.

Let a = the record of the N. and S. pen as it appears upon the plate.

b = the record of the E. and W. pen as it appears upon the plate.

d = distance from steady-point to point of support of pendulum.

e = distance from steady-point to point of pen.

$r = \frac{e}{d}$.

x = angle which the meridian of the pendulums makes with the true meridian of the place.

y and y' = angle between the direction of the pen-arm and a radius of the plate drawn through its point for the N. and S. and E. and W. pens, respectively.

z = angle which the radius of the plate drawn through the point of the pen makes with the true meridian of the place.

α = actual displacement of the Earth N. and S.

β = actual displacement of the Earth E. and W.

A.—The ratio of the pens is the ratio of the distances from the steady-point to the point of the pen, and from the steady-point to the point of support—in the instruments we are especially considering, the line joining the steel points which bear in the agate cups. Theoretically, the steady-point, or rather line, is the vertical line through the cylindrical weight about which the force of gravity is symmetrical. Practically, there is a little uncertainty as to the exact location of the steady-point—which, however, will be very near the axis of the cylindrical weight.

This ratio is given by the formula:—

$$r = \frac{e}{d}.$$

B.—The horizontal pendulums should be so adjusted that their meridian coincides with the true meridian of the place, *i. e.* that the plane (q) passing through the points of support and the steady-point of the pendulum, in the case of the E. and W. pendulum, should coincide with the meridian; in the N. and S. pendulum, this plane should lie E. and W.

If, however, there is no such coincidence, and the meridian of

the instruments makes an appreciable angle (x) with the true meridian, then the displacements of the pendulums in the true co-ordinates by the earthquake will vary with this angle. If the direction of the Earth's motion which it is designed to register is not *normal* to the plane (q), then the recorded motion will be less than it should be in the ratio of cos x : 1.

C.—If the horizontal pens are so situated that when at rest the radii of the plate passing through their points are tangent to the arcs described by them, then no factor is to be introduced on this account. Otherwise the displacement measured on such a radius will be too small in the proportion cos y : 1.

D.—The plate upon which the record is made is, of course, carried about by the Earth in its movements, which must be taken into account in deducing the actual motion of the Earth from the records of the pens.

In horizontal pendulums where the angle (p) between the lines drawn from the steady-point to the point of the pen, and from the steady-point to the point of support, is greater than ninety degrees, it can be shown that the *motion of the plate* due to the earthquake will be *additive* to the *pen's motion*, thus *increasing* the *record* of the pen, the plate being carried under the pen in an opposite direction to that in which the pen is moving. On the other hand, if the angle (p) is less than ninety degrees, the effect will be the opposite, *i. e.* to *decrease* the pen's record. This assumes that the pendulums are not *far* out of adjustment with respect to their meridians. In the Lick Observatory instruments the angle (p) is greater than ninety degrees, hence the effect is to *increase* the record. This is true for both co-ordinates.

The component motion of the *plate* N. and S. as projected on a radius depends upon the angle (z) which that particular radius makes with the meridian, and varies as the *cosine* of that angle.

The component motion of the *plate* E. and W. as projected on the radius passing through the point of the E. and W. pen will vary as the *sine* of the angle (z).

From the foregoing we deduce the following formulæ for the reduction of the observed records to the true displacements of the Earth:—

$$\frac{a}{\alpha} = \frac{e \cos x \sin y}{d} \pm \cos z, \qquad (1.)$$

$$\frac{b}{\beta} = \frac{e \cos x \sin y'}{d} \pm \sin z. \qquad (2.)$$

Professor SCHAEBERLE suggests that we may also consider the plate and supports of the pens as one rigid system, and the steady-point to move with respect to this system.

Let $f =$ distance from point of pen to point of support of pendulum, and, as before, $d =$ distance from steady-point to point of support of pendulum.

Then, on the above assumption, it can be shown that,

$$\frac{a}{\alpha} = \frac{b}{\beta} = \frac{f}{d}, \qquad (3.)$$

so long as the instrumental meridian coincides with the true meridian of the station, and the radius of the plate passing through the points of the pens is normal to the lines passing through the points of the pens and their points of support. If, however, the instrument is not in adjustment in these two particulars, due allowance must be made for such variations.

THE VERTICAL COMPONENT.

In the mechanism for recording the Earth's vertical motion, the pen proper is jointed to a vertical arm, which in turn is fastened rigidly to the counterpoised pendulum. The lifting by the Earth causes the joint between the pen-arm and the vertical arm to be displaced in the arc of a circle, whose center is the steady-point of the pendulum. This displacement is resolved into a *horizontal* component (s), which leads to the magnified record on the plate, and a *vertical* component (t).

Let $h =$ distance from steady-point to point where pen-arm is hinged to vertical arm.

$i =$ distance from point of support to hinge of pen-arm.

$j =$ distance from steady-point to point of support of pendulum.

$m =$ angular displacement of the hinge of pen-arm from the steady-point as a center.

$n =$ angle included between the lines drawn from pen-arm hinge to steady-point, and from pen-arm hinge to point of support of pendulum.

$s =$ horizontal component of the displacement of pen-arm hinge.

$t =$ vertical component of the displacement of pen-arm hinge.

$\gamma =$ vertical displacement of the Earth.

c = the record of the vertical pen as it appears upon the plate.

m and n are found from

$$\sin m = \frac{\gamma}{j}, \qquad (4.)$$

$$\tan n = \frac{j}{i}, \qquad (5.)$$

and we find s and t from

$$s = \frac{h \sin m}{\cos \frac{1}{2} m} \cos (\tfrac{1}{2} m + n), \qquad (6.)$$

$$t = \frac{h \sin m}{\cos \frac{1}{2} m} \sin (\tfrac{1}{2} m + n), \qquad (7.)$$

For ordinary displacements of the Earth (m being always small) we may write (6.) and (7.) in the following forms:—

$$s = h \sin m \cos n, \qquad (8.)$$

$$t = h \sin m \sin n, \qquad (9.)$$

It will be seen that the pen-arm hinge is lifted a little *higher* by the Earth's motion than the plate itself. This causes the pen's record on the plate to be *shortened* slightly.

In a seismograph of the usual form the dimensions are such that so long as the pen-arm makes but a small angle with the plane of the plate, this factor will be small.

To compute the amount of this shortening, we have the following quantities in a right triangle.

a' = distance from point of pen to hinge of pen-arm = hypothenuse.

b' = perpendicular let fall from hinge of pen-arm to plate.

c' = distance from pen's point to foot of perpendicular = base of triangle.

A', B', C' = angles opposite given sides respectively, A' being the right angle.

We find B' from

$$\sin B' = \frac{b'}{a'}, \qquad (10.)$$

and we have (approximately)

$$\Delta c' = - \frac{\cos C' \Delta b'}{\cos B'}, \qquad (11.)$$

in which $\Delta c'$ is the decrease in the *record* due to the increase ($\Delta b'$) in the distance from pen-arm hinge to plate as a result of the lifting of the instrument by the shock.

For the Lick Observatory instrument we have:—

$$a' = 5^{in}.75,$$
$$b' = 1^{in}.75,$$

Using this data, I have computed the shortening of the *record* due to this cause, and find it to be only $0^{in}.014$ for a vertical motion of the Earth of $0^{in}.50$. Hence it will be seen that for shocks likely to be observed with these instruments, this effect may be ignored without sensible error.

If in equation (8.) we substitute for h cos n its equivalent i, and for sin m its equivalent $\frac{\gamma}{j}$ we find (approximately),

$$\frac{s}{\gamma} = \frac{i}{j}.$$

It can be shown that the same result follows from considering the motion to be about the support of the pendulum as the axis.

Finally we have for the magnifying ratio of the vertical pen,

$$\frac{c}{\gamma} = \frac{s}{\gamma} + \Delta\ c', \qquad (12.)$$

in which $\Delta\ c'$ may be neglected, as shown, or with sufficient accuracy,

$$\frac{c}{\gamma} = \frac{i}{j}, \qquad (13.)$$

For the Lick Observatory instruments we have the following data:—

$$d = 3.75 \text{ inches},$$
$$e = 13.0 \text{ inches},$$
$$x = 6^{\circ},$$
$$y = 105^{\circ},$$
$$y' = 76^{\circ},$$
$$z = 38^{\circ}.5,$$
$$h = 10.3 \text{ inches},$$
$$i = 9.0 \text{ inches},$$
$$j = 5.0 \text{ inches},$$

from which we derive the following ratios:—

$$\frac{a}{\alpha} = 4.11, \qquad \text{(N. and S.)}$$

$$\frac{b}{\beta} = 3.97, \qquad \text{(E. and W.)}$$

$$\frac{c}{\gamma} = 1.8. \qquad \text{(Vertical.)}$$

The date given above and the constants deduced from them are suitable for the reduction of observations from April, 1893, to date.

MAGNIFYING RATIO OF THE DUPLEX SEISMOGRAPH.

In the ordinary form of this instrument there are two circumstances to be considered as affecting the magnification of the Earth's motion, viz:—

1st. The magnifying ratio of the vertical arm which is given by

$$\frac{a''}{b''},$$

in which

a'' = distance from lower end of vertical arm to level of glass plate;

b'' = distance from lower end of vertical arm to gimbal joint of bracket.

2d. The motion of the plate itself during the shock. It can be shown that the motion of the plate itself tends to decrease the *record* by the amount of the Earth's motion. Hence we have the following formula for the magnification:—

$$\frac{a'' - b''}{b''}, \qquad (14.)$$

In the Lick Observatory instrument of this class we have,

$$a'' = 13^{\text{in}}.10,$$
$$b'' = 2\;.35,$$

and consequently the magnifying ratio = 4.6.

Owing to uncertainties, such as the friction of the pen upon the plate, the friction of the pendulums at the point of support, the probable motion of the steady-point itself after a few seconds, and other minor causes, it is not necessary to take into account all the lesser factors affecting the magnification of the record. All that is here attempted is to include those which have a practical effect. I have not been able to find the formulæ for these reductions in any publication on the subject here.

MT. HAMILTON, CAL., March 14, 1898.